Journal of Engineering Mathematics **35:** 455–456, 1999.

Note on a paper by Riley and Yan

J. R. CHAPLIN[1], N. RILEY[2] and B. YAN[2]
[1]*Department of Civil Engineering, City University, London, EC1V 0HB, U.K.*
[2]*School of Mathematics, University of East Anglia, Norwich, NR4 7TJ, U.K.*

Received 6 January 1999; accepted 14 January 1999

Key words: wave propagation, free surface, submerged cylinder.

In a recent series of papers [1–3] Riley and Yan have studied the propagation of free-surface waves over a submerged circular cylinder whose generators are parallel to the wave crests. In [1] a perturbation solution for inviscid flow is developed in terms of the small parameter $\varepsilon = A/a$, where A is the wave amplitude and a the radius of the cylinder. They present, in particular, results in the form of instantaneous free-surface profiles, and time-averaged free-surface profiles. The latter shows a depression, or 'set-down' over the cylinder denoted by η_{20} which increases as the depth of the cylinder beneath the undisturbed free surface decreases. Chaplin has recently conducted a series of experiments in which he measures, *inter alia*, instantaneous free-surface profiles and their time average. The experiments are described in detail in [4]. We have thought it worthwhile to draw attention briefly to the relationship

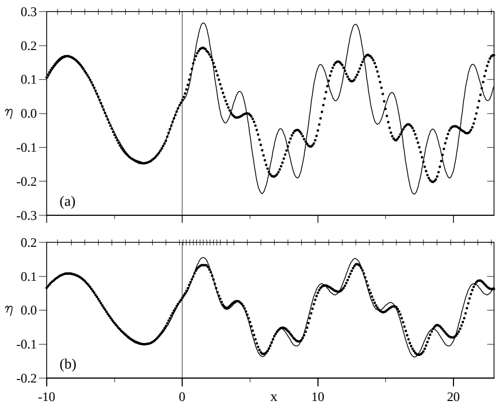

Figure 1. Measured and predicted instantaneous water surface profiles for $k = 0{\cdot}56, h = 1{\cdot}5$ at the instant at which the undisturbed wave would have a zero down-crossing at the location of the cylinder ($x = 0$). Measurements are shown as points, and the theoretical results as a line. The experiments were carried out in a wave tank that is 13 m long, 430 mm wide, with a still water depth of 700 mm. The instantaneous water-surface profile was accumulated from the first two temporal Fourier components of measurements made sequentially at the position identified by tick marks along the upper axis of each diagram. Intermediate points were interpolated by assuming a smooth variation with x of amplitude and phase at each frequency. Measured and predicted results for $\varepsilon = 0{\cdot}16$, the case computed by Vada [5] and Riley and Yan [1] are shown in (a). At this amplitude the waves broke shortly after passing over the cylinder. At $\varepsilon = 0{\cdot}107$ however, slightly below the breaking limit, Figure 1(b) shows excellent agreement.

between theory and this aspect of the experiments. We do this, in part, to demonstrate the excellent agreement between theory and experiment when ε is sufficiently small. However, whilst the theory is expected to be valid as long as the perturbation series converges, it is not possible to calculate the radius of convergence. On the other hand, the experiments clearly display an upper limit on ε for the theory, as evidenced by wave breaking. This manifests itself initially as a ripple in the otherwise smooth wave profile, and, although this is small, it is sufficient to alter the character of the wave that the theory is attempting to describe.

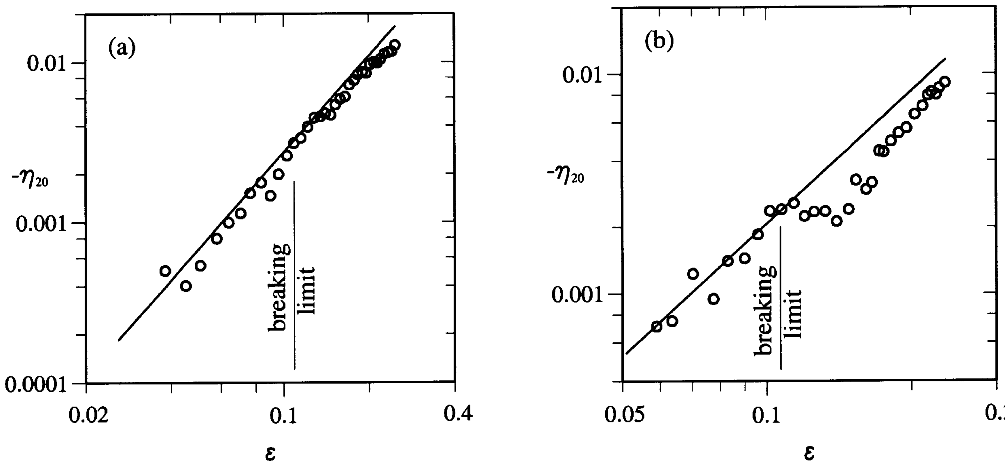

Figure 2. The variation, with ε, of the mean water surface elevation above the cylinder's axis, η_{20}. Measurements are shown as open circles and the theoretical results as a line. The breaking limit is identified. (a) $k = 0{\cdot}5$, $h = 1{\cdot}5$, (b) $k = 1$, $h = 1{\cdot}5$.

The comparison between theory and experiment is made in Figures 1 and 2. In Figure 1 instantaneous water-surface profiles are compared, whilst in Figure 2 we compare the experimentally obtained and theoretically predicted time-average surface profiles. Included in Figure 2 is the observed value of ε at which wave-breaking is first observed, and beyond which the theory is not valid.

Acknowledgment

The authors would like to thank the Marine Technology Directorate for financial support.

References

1. N. Riley and B. Yan, Inviscid flow around a submerged circular cylinder induced by free-surface travelling waves. *J. Engng. Math.* 30 (1996) 587–601.
2. B. Yan and N. Riley, Boundary-layer flow around a submerged circular cylinder induced by free-surface travelling waves. *J. Fluid Mech.* 316 (1996) 241–257.
3. B. Yan and N. Riley, Viscous flow about a submerged circular cylinder induced by free-surface travelling waves. *J. Fluid Mech.* 374 (1998) 173–194.
4. J. R. Chaplin, Non-linear wave interactions with a submerged horizontal cylinder. (1998). Paper in preparation.
5. T. A. Vada, A numerical solution of the second-order wave-diffraction problem for a submerged cylinder of arbitrary shape. *J. Fluid. Mech.* 174 (1987) 23–37.